Universality in the Large Signal Behaviour of Varactor Junctions

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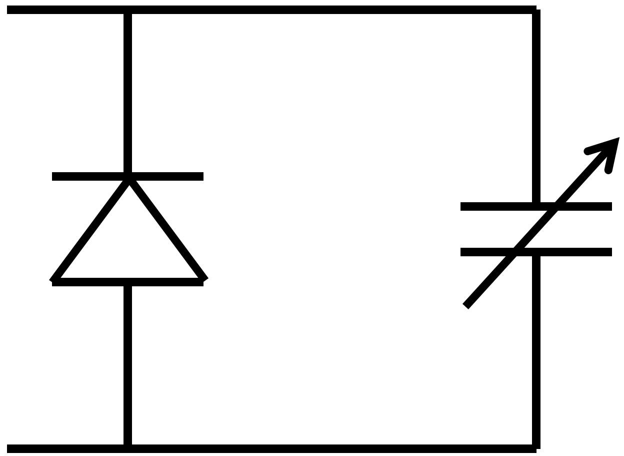
Semiconductor Varactors



T_1 = Time constant of carrier recombination

T_2 = Period of fundamental signal

CASE A

$T_1 << T_2$

CASE B

$T_1 >> T_2$

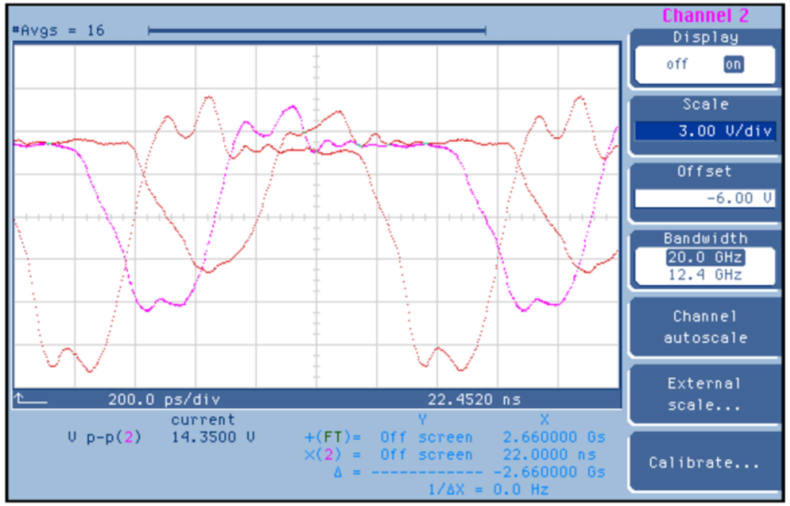
Conjectures

CASE A

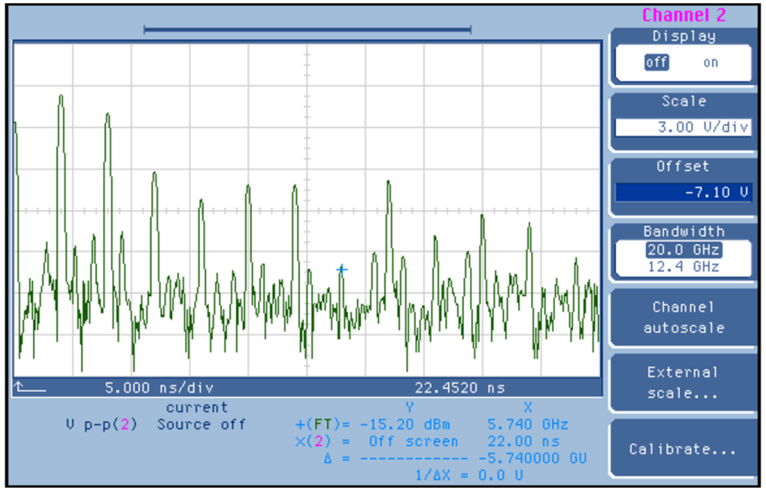
Within the constraints of the network within which it is embedded, for a given incident power level supplied at the fundamental frequency, the DC voltage developed across the varactors will always be minimised.

CASE A

VOLTAGE



SPECTRUM



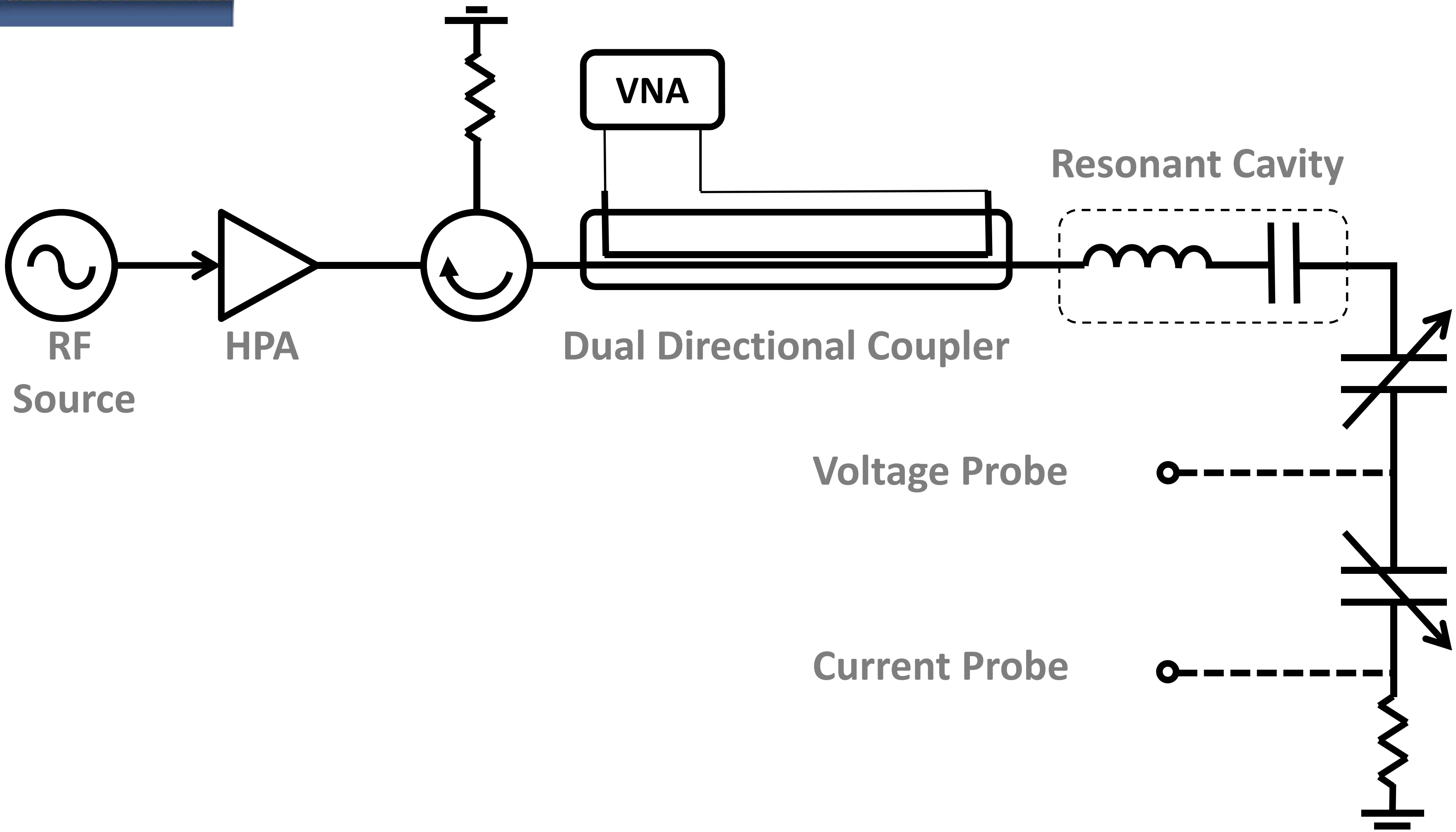
Pin – relative dBm	CW Frequency MHz	Fundamental dBm	F2 dBm	F3 dBm	F4 dBm	F5 dBm	F6 dBm	F7 dBm	File
-14	806	21.3	17.6	3.8	-5.4	-5.8	-8	0	I587
-12.5	811	24.1	20.0	6.6	-2.3	-0.2	+2	+1.5	I589
-11	822.85	26.4	21.8	7.3	+2.0	+5.5	+4.0	-15	I590

Measured voltage of spectrum and gate circuit of a GaAs pHEMT with open circuits at harmonics

$$V_{DC} = \frac{V_{RF}}{2\cos\left(\frac{\pi}{n+2}\right)} \quad with\, zeros\, at\, \theta_r = sin\left[\frac{(2r-1)\pi}{n+2}\right] \quad where\, r = 1 \rightarrow n$$

$$V_{max} = \frac{2\cot^2\left(\frac{\pi}{2(n+2)}\right)}{n+2}.V_{DC}$$

Case B: Experimental Setup



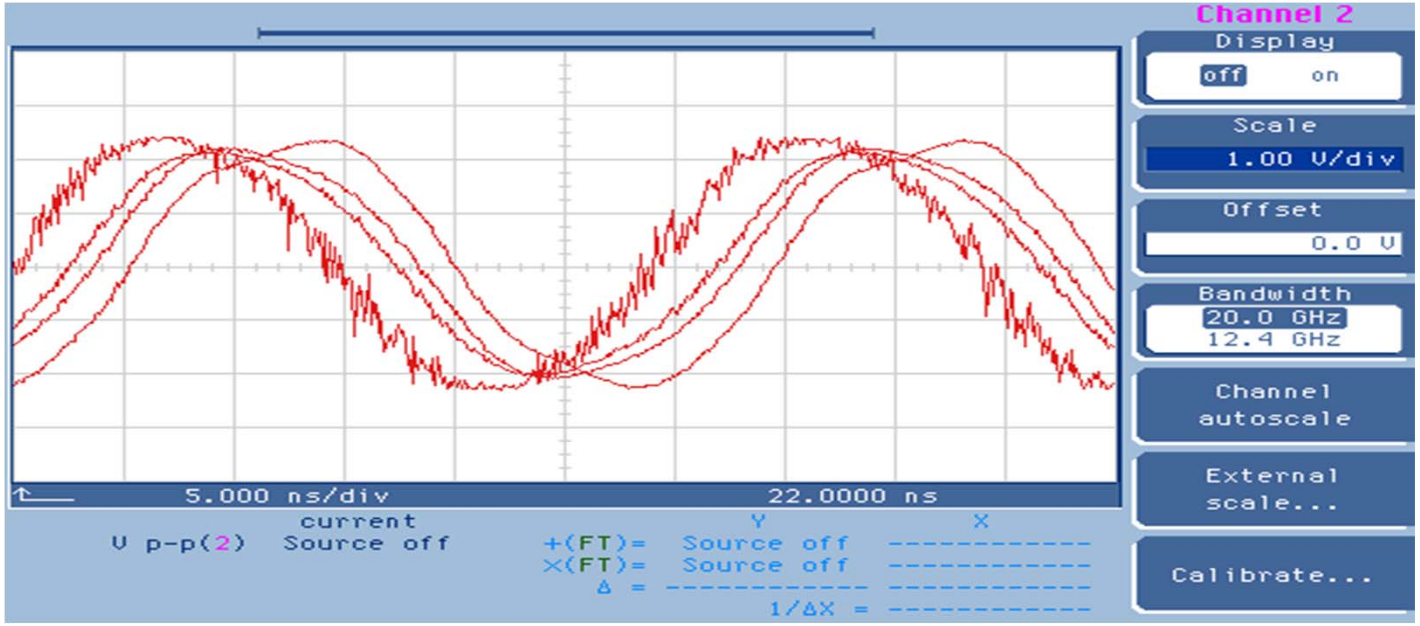
Conjectures

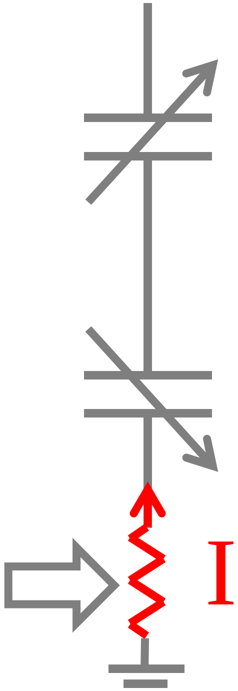
CASE B

Within the constraints of the network within which it is embedded, for a given incident power level supplied at the fundamental frequency, the current limits and the amount of sub-harmonic energy oscillating between the varactors will always be maximised.

CASE B: PN Junction Varactors
Type 1 : Abrupt Junction

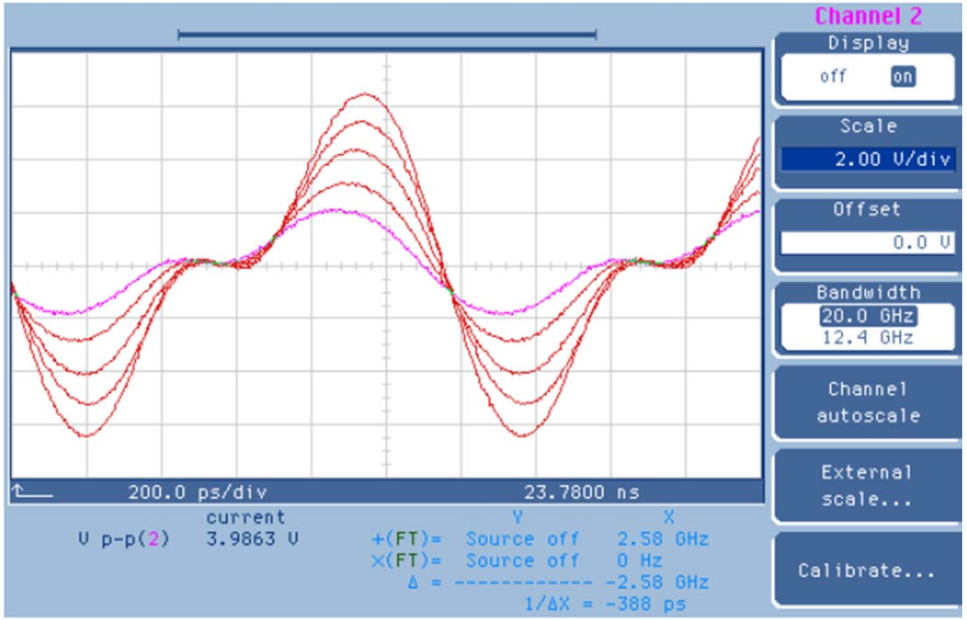
Current measurement at varying power levels



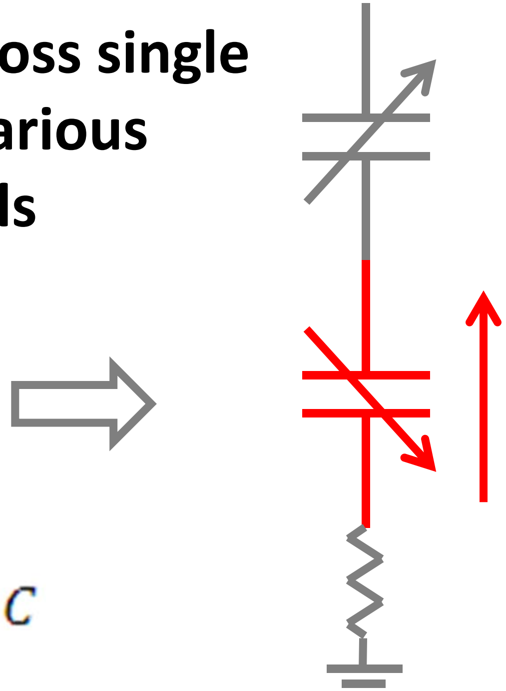


Pin	Pdc (centre tap voltage)	f0(dBm)	2f0(dBm)	3f0(dBm)	4f0(dBm)	5f0(dBm)
4.5	12.9	22.5	-22	.5	-15	-10.2
2.45	11.4	22.3	-20	-.5	-40	-40
1.14	6.3	22.1	-20	-6	-40	-40
0.44	2.1	21.8	-40	-4.5	-40	-40

CASE B: PN Junction Varactors
Type 1 : Abrupt Junction



Voltage across single device at various power levels



$$V_{pp} = V_{DC}$$

dBc	Volts	dBc			
Pin	Vdc	f0(dBm)	2f0(dBm)	3f0(dBm)	4f0(dBm)
8.3	12.3	22.8	18.3	-0.5	-7
6.36	11.6	22.65	17.0	0	-9
4.0	9.16	20.25	13.5	-2.5	-8.7
2.2	6.33	16.7	10.0	-5.6	-11.3
1.16	4.2	12.8	6.2	-9.8	-14

Instantaneous Power

$P = Instantaneous\ Power\ Extracted\ from\ bottom\ device$

$$P \propto -sin\theta cos\theta(1 + \varphi\ cos\theta)$$

Where

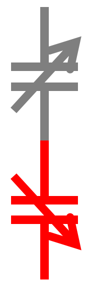
$$I \propto -cos\theta$$

$$V \propto sin\theta(1 + \varphi\ cos\theta)$$

Instantaneous Energy

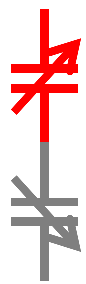
Bottom Device

$$E \propto \frac{cos^2\theta}{2}\left(1+\frac{2\varphi}{3}\,cos\theta\right) \geq 0$$



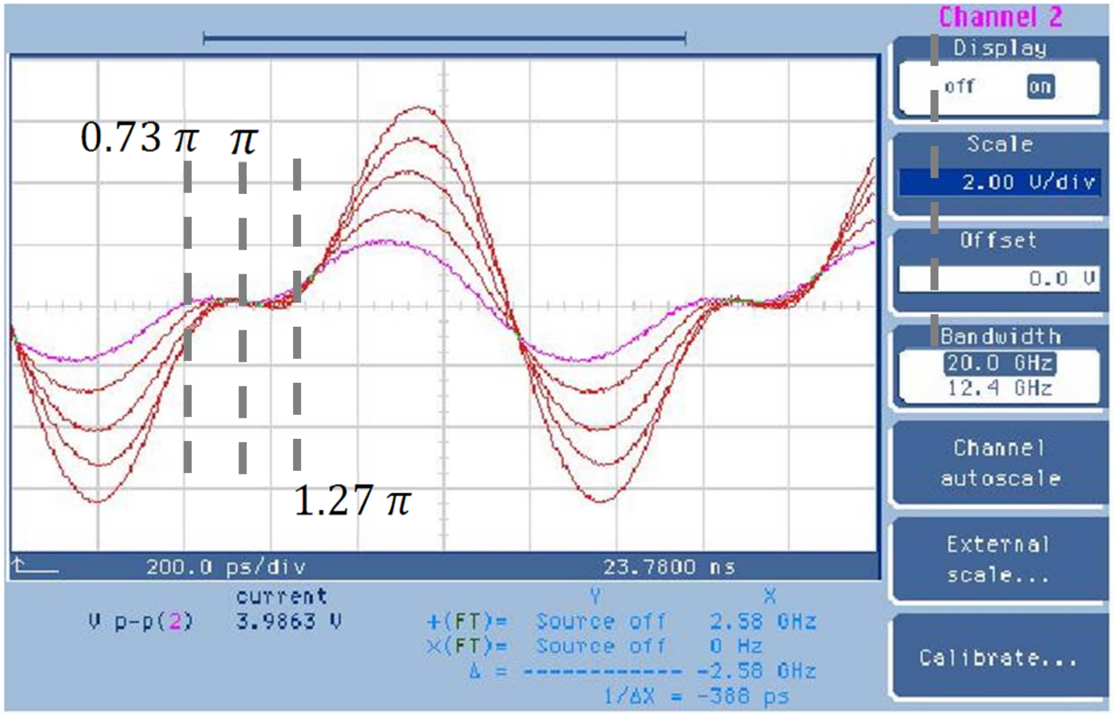
Top Device

$$E \propto \frac{cos^2\theta}{2}\left(1-\frac{2\varphi}{3}\,cos\theta\right) \geq 0$$



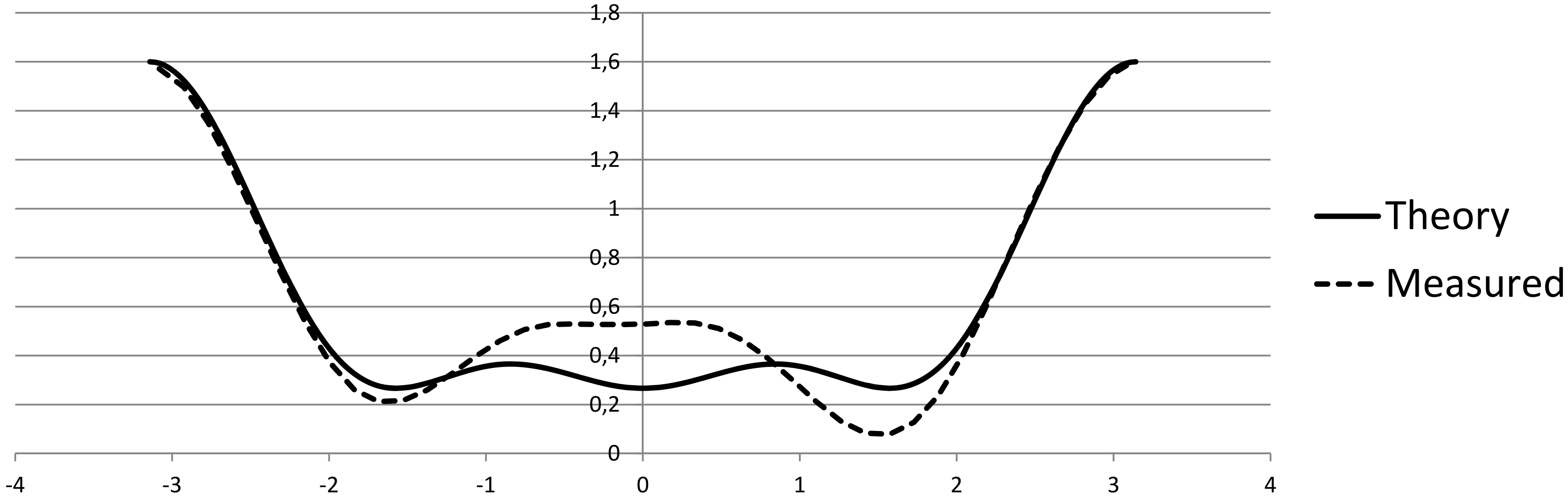
Voltage

$$\varphi = \frac{3}{2} \qquad \therefore\ voltage\ zero\ at\ \theta = cos^{-1}\left(-\frac{2}{3}\right)$$



Energy Function

$$E \propto \frac{cos^2\theta}{2}(1 + cos\theta)$$



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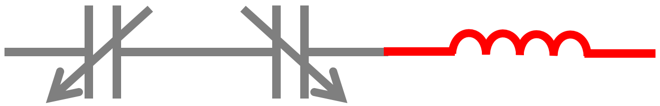
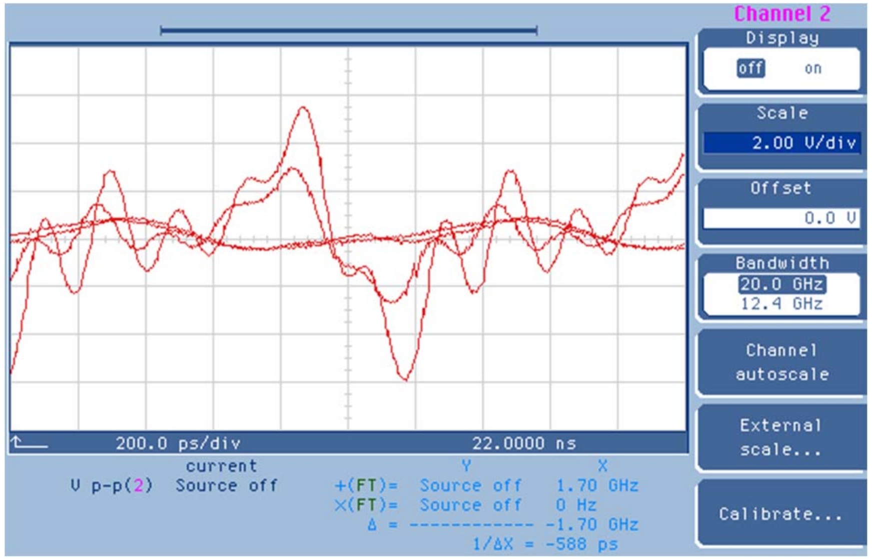
CASE B: PN Junction Varactors
Type 2 : Hyper - Abrupt Junction

Initially, additional inductance was added in series with the devices to ensure that no third harmonic current could flow.





The measured current was approximately sinusoidal and the measured voltage was

$$V_{pp} = V_{DC}$$

Pin	Pdc	f0	2f0	3f0	4f0	5f0	6f0
3.7	11.1	15.0	16.9	4.8	-6.7	6.2	13.3
2.12	5.7	2.0	9.4	-6.3	-4.4	-13.0	0
1.68	2.58	1.8	-1.0	-	-23	-17	-
1.11	1.53	2.85	-7	-	-	-	-

CASE B: PN Junction Varactors Type 2 : Hyper - Abrupt Junction

Hence, the energy function will be of 4th degree of the form:

$$E \; \propto \; 1 + A\,cos\theta + B\;cos2\theta + C\;cos3\theta + D\;cos4\theta$$

If this is zero with turning points at $\theta_1, \theta_{2,} 2\pi - \theta_1 \;\; 2\pi - \theta_2$ *and chosen such that A is maximised, the solution is* ***singular***!

CASE B: PN Junction Varactors
Type 2 : Hyper - Abrupt Junction

$Let\ \theta_1 = \frac{\pi}{2}\ and\ \theta_2 = \frac{5\pi}{6}, then$

$\theta = \frac{\pi}{2} \qquad 1 - B + D = 0 \qquad Equation\ 1$

$\theta = \frac{5\pi}{6} \qquad 1 - \frac{\sqrt{3}}{2}A + \frac{B}{2} - \frac{D}{2} = 0 \qquad Equation\ 2$

Dividing Equation 1 by 2 and adding to Equation 2 gives
$A = \sqrt{3}$ *which is the stationary maximum solution*

Hence,

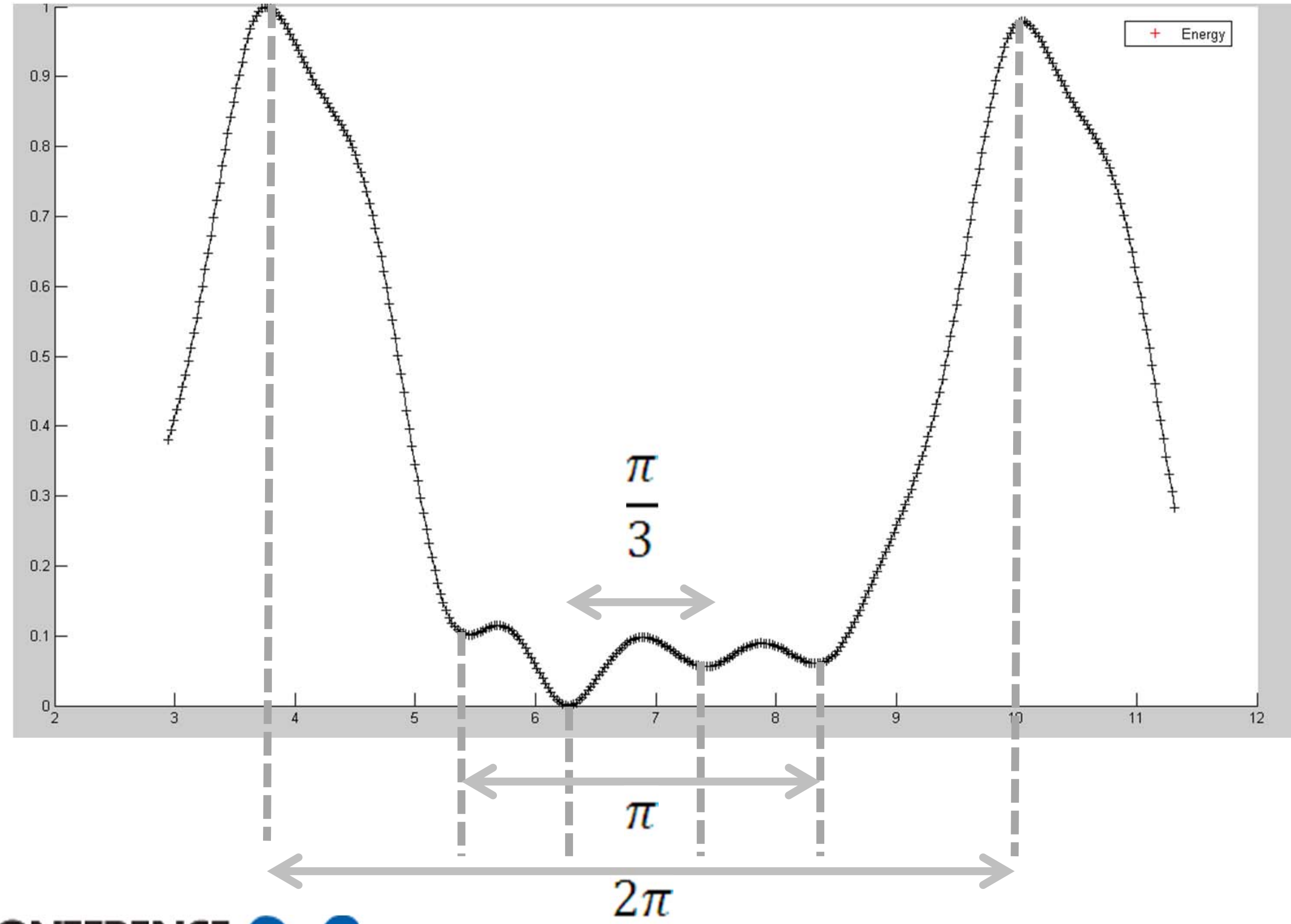
$$E \propto cos^2\theta \left(\frac{\sqrt{3}}{2} + cos\theta)\right)^2$$

$with\ I \propto -cos\theta$

$$V \propto \frac{3}{2}\sin\theta + \frac{3\sqrt{3}}{2}\sin 2\theta - \sin 3\theta$$

CASE B: PN Junction Varactors Type 2 : Hyper - Abrupt Junction

...and the measured value of energy is :



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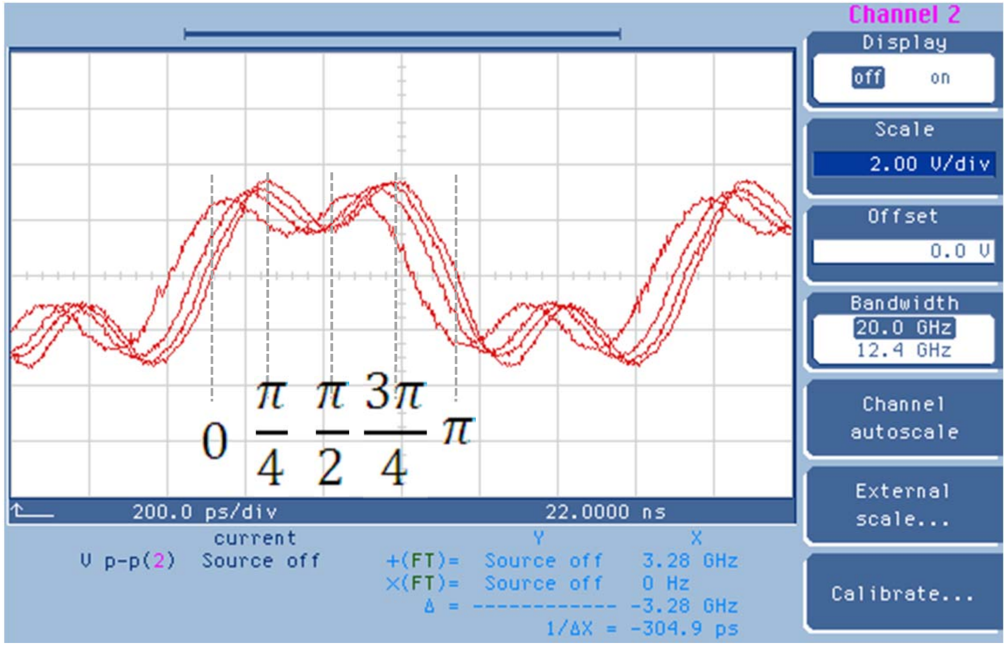
Hyper Abrupt with Finite Reactance at the 3rd Harmonic

Removing the series inductor produces a measured current of:



<u>Constant current at different power levels of 0,-3,-6,-12 dB</u>

Hyper Abrupt with Finite Reactance at the 3rd Harmonic

Hence,

$$I \propto -\left(\cos\theta - \frac{1}{3}cos3\theta\right)$$

$$\frac{dI}{d\theta} = 0 \ at \ cos^2\theta = \frac{1}{2}$$

Thus, $E_V.E$ *may be expressed as :*

$$Even\ (E) \propto (1 + A\cos 2\theta)^2\ (6A + \ 1 - 2A\cos 2\theta)$$

with,

$$\frac{dE}{d\theta} = P \propto \ -12A^2 \sin 2\theta(1 + A\cos 2\theta)(2 - \cos 2\theta)$$

Hyper Abrupt with Finite Reactance at the 3rd Harmonic

with,

$$I \propto -\cos\theta\,(2-\cos 2\theta) = -\frac{3}{2}\left(\cos\theta - \frac{1}{3}\cos 3\theta\right)$$

and

$$V \propto \sin\theta\,(1 + \mathrm{A}\cos 2\theta)$$

$$= \left(1 - \frac{A}{2}\right)\sin\theta + \frac{A}{2}\sin 3\theta$$

Hyper Abrupt with Finite Reactance at the 3rd Harmonic

Hence,

Input energy at fundamental frequency $\propto A^2\left(1-\frac{A}{2}\right) = E_1$

and

Output energy from 3rd harmonic termination $\propto \frac{A^3}{6} = E_3$

Energy in lower device is :

$$E \propto (1 + A\cos 2\theta)^2\,(6A + 1 - 2A\cos 2\theta) + K + \varphi\, cos^3\theta\left(1 - \frac{2}{5}cos^5\theta\right) \geq 0$$

Hyper Abrupt with Finite Reactance at the 3rd Harmonic

$$P \propto -\sin 2\theta \,(2 - \cos 2\theta)\left(12\,A^2(1 + A\cos 2\theta) + \frac{\varphi}{2}\cos\theta\right)$$

$If\ zero\ at\ \theta = \theta_0, then$

$$\varphi = \frac{-24A^2(1 + A\cos 2\theta_0)}{\cos\theta_0}$$

$To\ be\ maximum\ and\ stationary, \varphi\ must\ have\ the\ same\ dependence\ as\ E_1$

$$Hence\ \theta_0 = \frac{2\pi}{3}\ and$$

$$\varphi = 48\,A^2\left(1 - \frac{A}{2}\right)$$

$$For\ E\ to\ be\ zero\ at\ \theta = \theta_0$$

$$K = \frac{-89\,A^3 + 243\,A^2 - 120\,A - 20}{20}$$

$$and\ is\ a\ stationary\ when$$

$$A = 1.5256$$

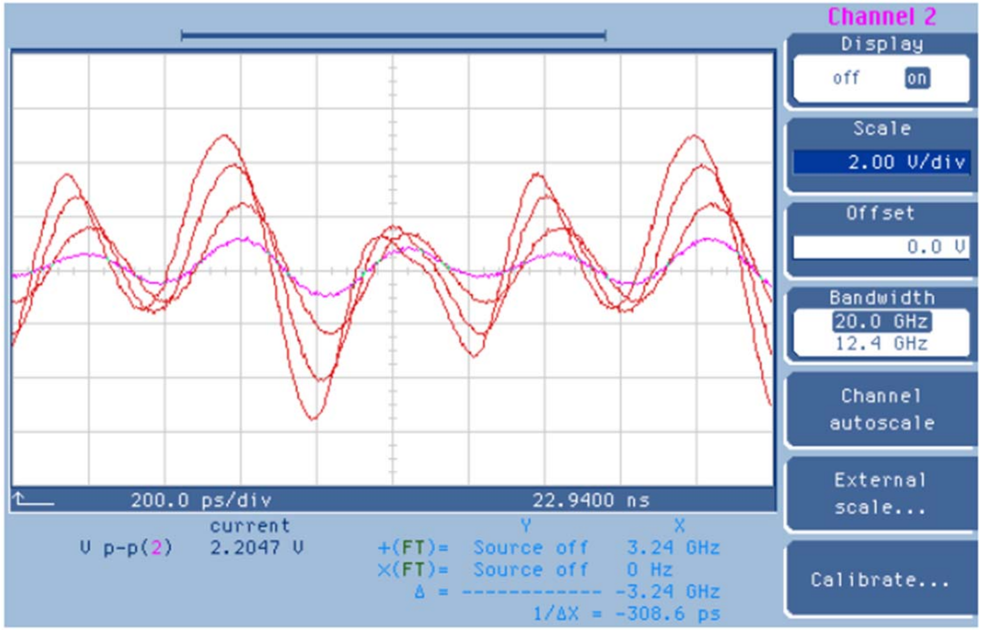
$$Hence$$

$$V \propto \left(1 - \frac{A}{2}\right)\sin\theta + \left(1 - \frac{A}{2}\right)\sin 2\theta + \frac{A}{2}\sin 3\theta$$

$$\approx \propto\ \sin\theta + \sin 2\theta + 3.2\sin 3\theta$$

Hyper Abrupt with Finite Reactance at the 3rd Harmonic

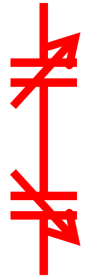
Measured Voltage at Varying Power Levels



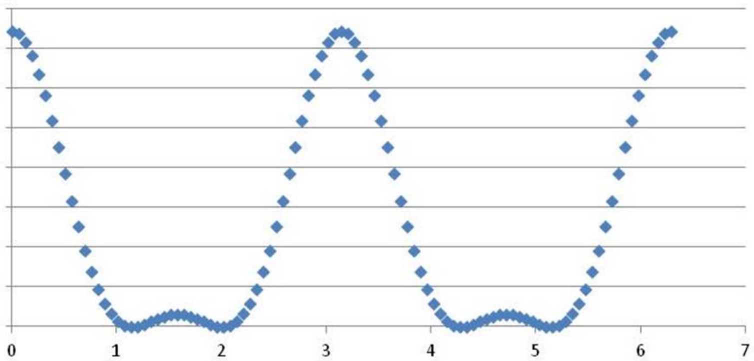
dBc	Volts	dBc				
Pin	Vdc	f0	2f0	3f0	4f0	5f0
9.0	7.6	12.3	10.2	19.15	-40	1
5.07	4.8	7.9	7.7	18.0	-40	-2.8
2.93	3.8	2.9	4.8	14.9	-20	-18
1.47	2.6	-13.9	0	5.9	-20	-23

$$V_{pp} = V_{DC}$$

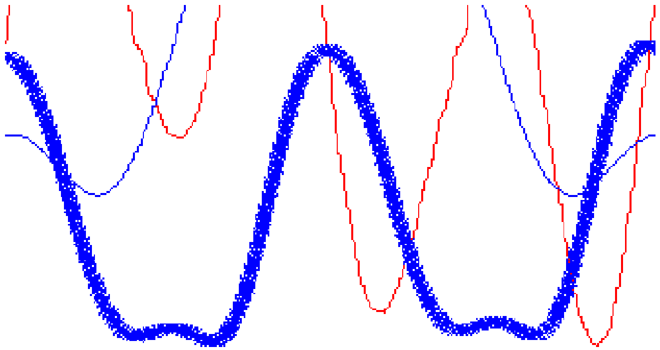
Energy Function in both Varactors



THEORY

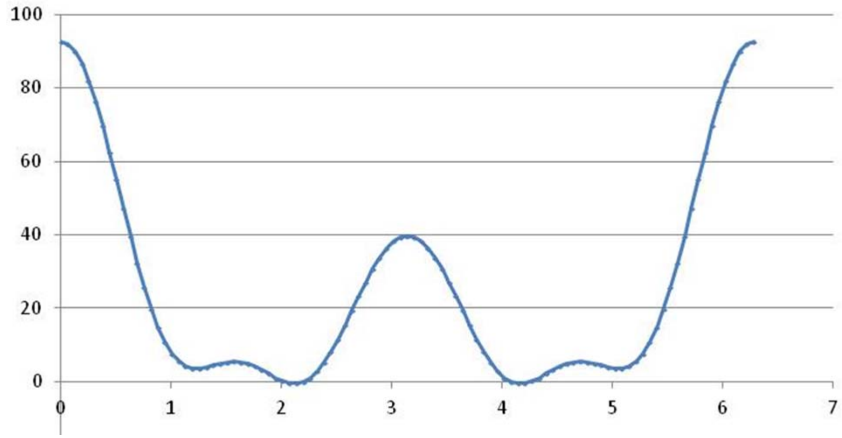


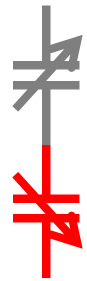
MEASURED



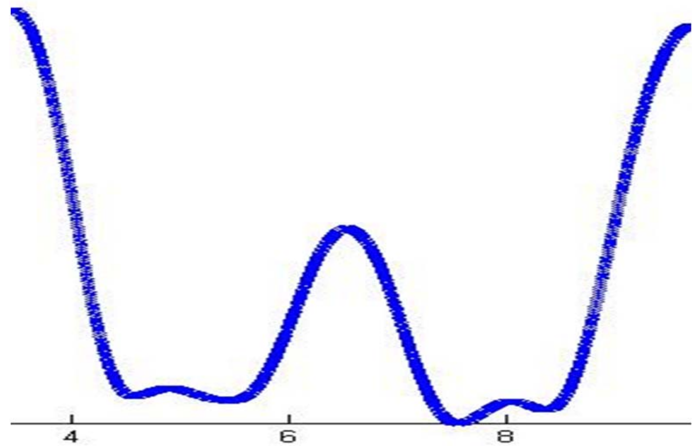
Comparison of Theoretical and Measured – Single Device

THEORY

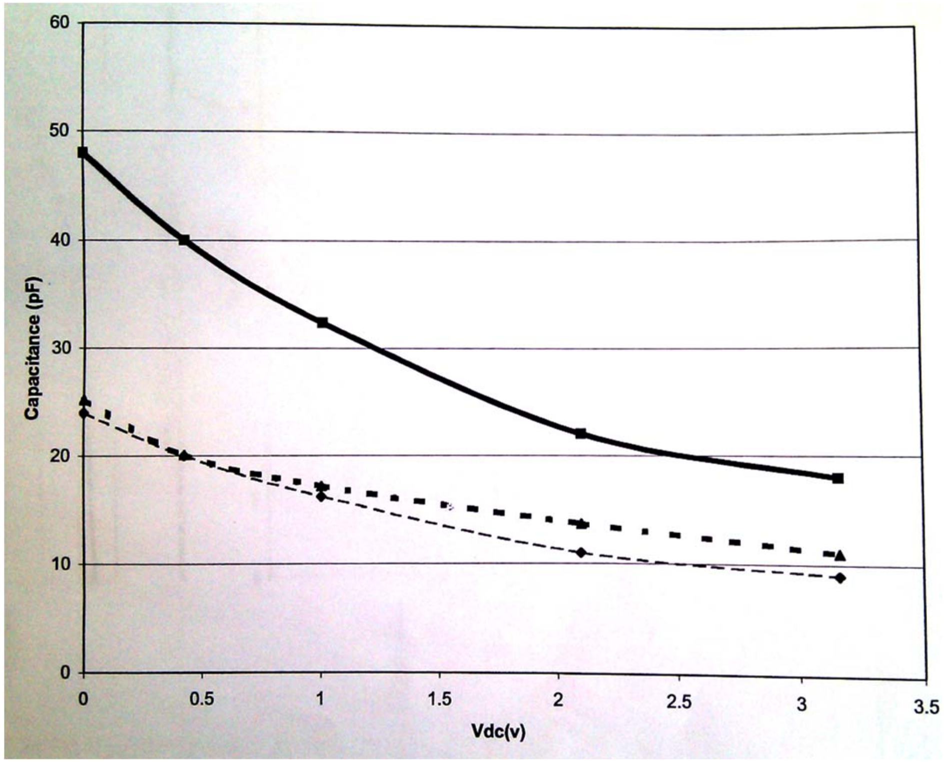




MEASURED

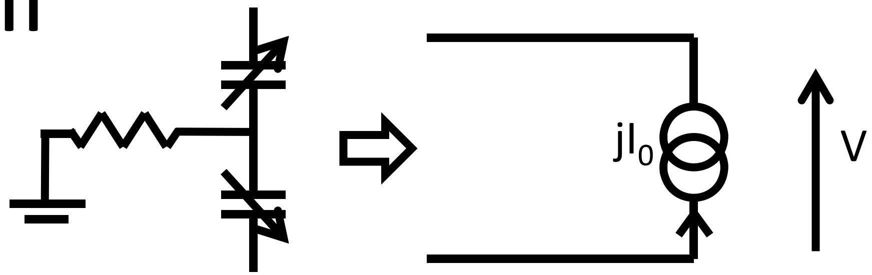


Measured Effective Capacitance at Fundamental



Third harmonic energy stored in external reactance
≈ energy stored at fundamental in devices

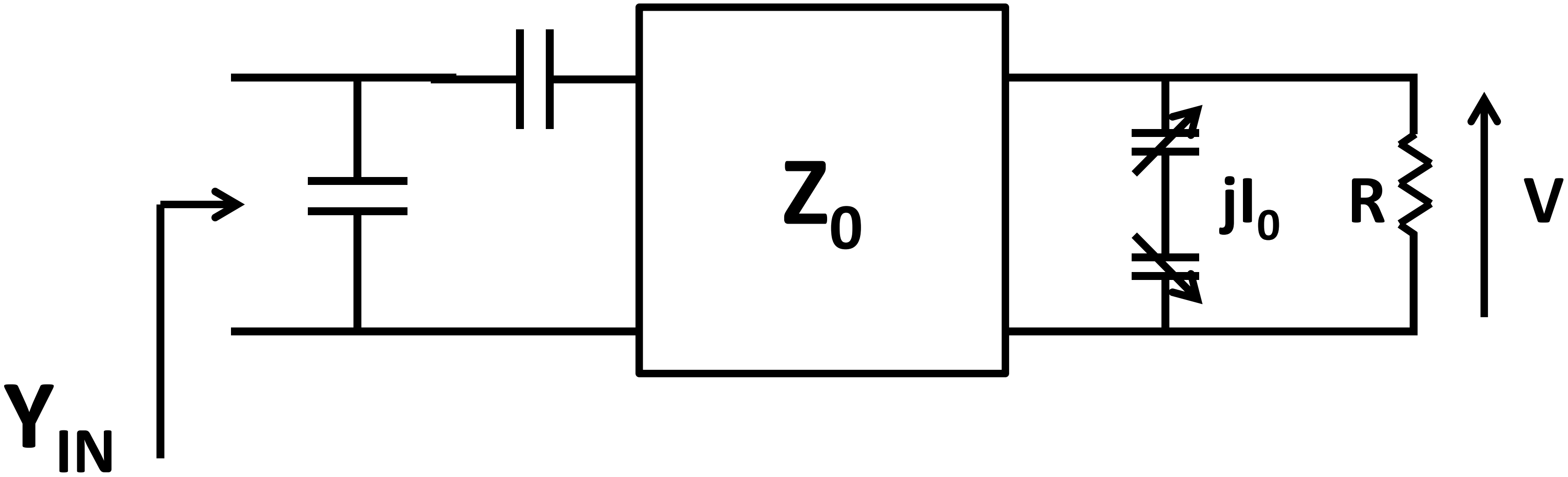
Equivalent Circuit of Varactor Pair



- **Constant current generator**
  - Constrained such that voltage is always at 90° to the constant current generator

- N.B. Some resistance has to be added at the common junction to discharge the DC voltage at > modulation rate of the signals

Typical Application for PA



Output circuit for power amplifier producing an approximately constant level of voltage at the input to provide high efficiency over a high dynamic range

Conclusion 1

For large signals across varactor diodes, the non-linear equation

$$I = C(V)\,\frac{dV}{dt}$$

Is **NOT** valid

Conclusion 2

For PN junction devices, over many RF cycles, the behaviour is totally governed by the sub-harmonic energy oscillating between the back to back devices which is maximised within the constraint of the network in which it is embedded.

Conclusion 3

Assuming discharge is accomplished at the signal modulation rate, the equivalent circuit is simply a current generator constrained by the phase of the voltage across it always being at 90^{o} to the current.

Conclusion4

Opens up a whole new field in non-linear circuit theory

Acknowledgements

The author would like to thank Dr. Mike Roberts of Slipstream Design for all the experimental work which was difficult, and for the preparation of these slides